

Model Traditional Pathway: Model Algebra I Overview [AI]

Number and Quantity

The Real Number System

- A. Extend the properties of exponents to rational exponents.
- B. Use properties of rational and irrational numbers.

Quantities

- A. Reason quantitatively and use units to solve problems.

Algebra

Seeing Structure in Expressions

- A. Interpret the structure of linear, quadratic, and exponential expressions with integer exponents.
- B. Write expressions in equivalent forms to solve problems.

Arithmetic with Polynomials and Rational Expressions

- A. Perform arithmetic operations on polynomials.

Creating Equations

- A. Create equations that describe numbers or relationships.

Reasoning with Equations and Inequalities

- A. Understand solving equations as a process of reasoning and explain the reasoning.
- B. Solve equations and inequalities in one variable.
- C. Solve systems of equations.
- D. Represent and solve equations and inequalities graphically.

Functions

Interpreting Functions

- A. Understand the concept of a function and use function notation.
- B. Interpret linear, quadratic, and exponential functions with integer exponents that arise in applications in terms of the context.
- C. Analyze functions using different representations.

Building Functions

- A. Build a function that models a relationship between two quantities.
- B. Build new functions from existing functions.

Linear, Quadratic, and Exponential Models

- A. Construct and compare linear, quadratic, and exponential models and solve problems.
- B. Interpret expressions for functions in terms of the situation they model.

Statistics and Probability

Interpreting Categorical and Quantitative Data

- A. Summarize, represent, and interpret data on a single count or measurement variable. Use calculators, spreadsheets, and other technology as appropriate.
- B. Summarize, represent, and interpret data on two categorical and quantitative variables.
- C. Interpret linear models.

Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Model Traditional Pathway: Model Algebra I Content Standards [AI]

Number and Quantity

The Real Number System

AI.N-RN

A. Extend the properties of exponents to rational exponents.

1. Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents.
For example, we define $5^{1/3}$ to be the cube root of 5 because we want $(5^{1/3})^3 = 5^{(1/3)^3}$ to hold, so $(5^{1/3})^3$ must equal 5.
2. Rewrite expressions involving radicals and rational exponents using the properties of exponents.

B. Use properties of rational and irrational numbers.

3. Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.

Quantities

AI.N-Q

A. Reason quantitatively and use units to solve problems.

1. Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays. ★
2. Define appropriate quantities for the purpose of descriptive modeling. ★
3. Choose a level of accuracy appropriate to limitations on measurement when reporting quantities. ★

Algebra

Seeing Structure in Expressions

AI.A-SSE

A. Interpret the structure of linear, quadratic, and exponential expressions with integer exponents.

1. Interpret expressions that represent a quantity in terms of its context. ★
 - a. Interpret parts of an expression, such as terms, factors, and coefficients.
 - b. Interpret complicated expressions by viewing one or more of their parts as a single entity.

For example, interpret $P(1+r)^t$ as the product of P and a factor not depending on P .

2. Use the structure of an expression to identify ways to rewrite it.

For example, see $(x+2)^2 - 9$ as a difference of squares that can be factored as $((x+2)+3)((x+2)-3)$.

B. Write expressions in equivalent forms to solve problems.

3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.
 - a. Factor a quadratic expression to reveal the zeros of the function it defines.
 - b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.
 - c. Use the properties of exponents to transform expressions for exponential functions.

For example, the expression 1.15^t can be rewritten as $(1.15^{1/12})^{12t} \approx 1.012^{12t}$ to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.

Arithmetic with Polynomials and Rational Expressions

AI.A-APR

A. Perform arithmetic operations on polynomials.

1. Understand that polynomials form a system analogous to the integers, namely, they are closed under certain operations.

- a. Perform operations on polynomial expressions (addition, subtraction, multiplication), and compare the system of polynomials to the system of integers when performing operations.
- b. Factor and/or expand polynomial expressions, identify and combine like terms, and apply the Distributive property.

Creating Equations

AI.A-CED

A. Create equations that describe numbers or relationships.

1. Create equations and inequalities in one variable and use them to solve problems. (Include equations arising from linear, quadratic, and exponential functions with integer exponents.) ★
2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. ★
3. Represent constraints by linear equations or inequalities, and by systems of linear equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. ★

For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.

4. Rearrange formulas to highlight a quantity of interest using the same reasoning as in solving equations (Properties of equality). ★

For example, rearrange Ohm's law $R = \frac{V^2}{P}$ to solve for voltage, V . Manipulate variables in formulas used in financial contexts such as for simple interest, $I = Prt$.

Reasoning with Equations and Inequalities

AI.A-REI

A. Understand solving equations as a process of reasoning and explain the reasoning.

1. Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify or refute a solution method.

B. Solve equations and inequalities in one variable.

3. Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.
 - a. Solve linear equations and inequalities in one variable involving absolute value.
4. Solve quadratic equations in one variable.
 - a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.
 - b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula, and factoring, as appropriate to the initial form of the equation. Recognize when the solutions of a quadratic equation results in non-real solutions and write them as $a \pm bi$ for real numbers a and b .

C. Solve systems of equations.

5. Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.
6. Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.
7. Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically.

For example, find the points of intersection between the line $y = -3x$ and the circle $x^2 + y^2 = 3$.

D. Represent and solve equations and inequalities graphically.

10. Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line). Show that any point on the graph of an equation in two variables is a solution to the equation.
11. Explain why the x -coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using

technology to graph the functions and make tables of values. Include cases where $f(x)$ and/or $g(x)$ are linear and exponential functions. ★

12. Graph the solutions of a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set of a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

Functions

Interpreting Functions

AI.F-IF

A. Understand the concept of a function and use function notation.

1. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output (range) of f corresponding to the input x . The graph of f is the graph of the equation $y = f(x)$.
2. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

For example, given a function representing a car loan, determine the balance of the loan at different points in time.

3. Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers.

For example, the Fibonacci sequence is defined recursively by $f(0) = f(1) = 1$, $f(n + 1) = f(n) + f(n - 1)$ for $n \geq 1$.

B. Interpret linear, quadratic, and exponential functions with integer exponents that arise in applications in terms of the context.

4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; and end behavior. ★
5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. ★

For example, if the function $h(n)$ gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.

6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. ★

C. Analyze functions using different representations.

7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. ★
 - a. Graph linear and quadratic functions and show intercepts, maxima, and minima. ★
 - b. Graph piecewise-defined functions, including step functions and absolute value functions. ★
 - e. Graph exponential functions showing intercepts and end behavior. ★
8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
 - a. Use the process of factoring and completing the square in a quadratic function to show zeros, maximum/minimum values, and symmetry of the graph, and interpret these in terms of a context.
 - b. Use the properties of exponents to interpret expressions for exponential functions. Apply to financial situations such as identifying appreciation and depreciation rate for the value of a house or car some time after its initial purchase: $V_n = P(1+r)^n$.

For example, identify percent rate of change in functions such as $y = (1.02)^t$, $y = (0.97)^t$, $y = (1.01)^{12t}$, and $y = (1.2)^{t/10}$, and classify them as representing exponential growth or decay.

9. Translate among different representations of functions (algebraically, graphically, numerically in tables, or by verbal descriptions). Compare properties of two functions each represented in a different way.

For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.

Building Functions

AI.F-BF

A. Build a function that models a relationship between two quantities.

1. Write linear, quadratic, and exponential functions that describe a relationship between two quantities. ★
 - a. Determine an explicit expression, a recursive process, or steps for calculation from a context. ★
 - b. Combine standard function types using arithmetic operations. ★

For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.

2. Write arithmetic and geometric sequences both recursively and with an explicit formula them to model situations, and translate between the two forms. ★

B. Build new functions from existing functions.

3. Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $kf(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Include linear, quadratic, exponential, and absolute value functions. Utilize technology to experiment with cases and illustrate an explanation of the effects on the graph.
4. Find inverse functions algebraically and graphically.
 - a. Solve an equation of the form $f(x) = c$ for a linear function f that has an inverse and write an expression for the inverse.

Linear, Quadratic, and Exponential Models

AI.F-LE

A. Construct and compare linear, quadratic, and exponential models and solve problems.

1. Distinguish between situations that can be modeled with linear functions and with exponential functions. ★
 - a. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals. ★
 - b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. ★
 - c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another. ★
2. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (including reading these from a table). ★
3. Observe, using graphs and tables, that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function. ★

B. Interpret expressions for functions in terms of the situation they model.

5. Interpret the parameters in a linear or exponential function (of the form $f(x) = b^x + k$) in terms of a context. ★

Statistics and Probability

Interpreting Categorical and Quantitative Data

AI.S-ID

A. Summarize, represent, and interpret data on a single count or measurement variable. Use calculators, spreadsheets, and other technology as appropriate.

1. Represent data with plots on the real number line (dot plots, histograms, and box plots). ★
2. Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets. ★
3. Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers). ★

B. Summarize, represent, and interpret data on two categorical and quantitative variables.

5. Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data. ★
6. Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. ★
 - a. Fit a linear function to the data and use the fitted function to solve problems in the context of the data. Use functions fitted to data or choose a function suggested by the context (emphasize linear and exponential models).
 - b. Informally assess the fit of a function by plotting and analyzing residuals. ★
 - c. Fit a linear function for a scatter plot that suggests a linear association. ★

C. Interpret linear models.

7. Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data. ★
8. Compute (using technology) and interpret the correlation coefficient of a linear fit. ★
9. Distinguish between correlation and causation.